RESEARCH DEPARTMENT

The Effect of the Ground Constants, and of an Earth System, on Radiation from a Vertical Aerial

Report No. E.049

Serial No. 1954/19

THE BRITISH BROADCASTING CORPORATION
ENGINEERING DIVISION

RESEARCH DEPARTMENT

THE EFFECT OF THE GROUND CONSTANTS, AND OF AN EARTH SYSTEM, ON RADIATION FROM A VERTICAL AERIAL

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LIST OF PRINCIPAL SYMBOLS

(Symbols used only in the Appendices are excluded)

	The complex electric field strength (V/m) at some distant point for an input current I_0:
E_1	for perfectly-conducting ground
E	for imperfectly-conducting ground
E'	for imperfectly-conducting ground with an earth system.
$E_1(\theta)$, $E(\theta)$, $E'(\theta)$	Correspond to E_1, E, E', being the field at an angle θ to the vertical.
$e^{j\omega t}$	Time factor (suppressed from equations).
F_{AB}	Groundwave attenuation factor between A and B (defined by Equation 8).
$F(x)$	Function defined by Equation 5.
h	Effective height of a vertical aerial (m), (equals moment divided by input current).
I_0	Current impressed between the terminals of an aerial (amps).
	Total surface current (amps) at radius r for an input current I_0:
$I_1(r)$	for perfectly-conducting ground*
$I(r)$	for imperfectly-conducting ground*
$I'(r)$	for imperfectly-conducting ground with an earth system.
$J_n(x)$	Bessel function of the first kind, order n.
$J_c(\mu, x)$, $J_s(\mu, x)$	Functions defined by Equation 12.
$K(r_0)$	Factor correcting for the effect of the induction field on the total surface current at radius r_0, defined by Equation 13.
r	Distance of a point on the ground from the base of the aerial (m).
r_0	Radius of earth system (m).

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*But see footnote to Appendix 7.4.

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LIST OF PRINCIPAL SYMBOLS

(cont'd)

t	Time (seconds).
$\mathfrak{z}$	Complex surface impedance of the ground (ohms).
$\mathfrak{z}'$	Surface impedance of the ground in the presence of the earth system.
β	$2\pi/\lambda$.
γ	Efficiency (defined by Equation 7).
η	Intrinsic impedance of free space (= 120π ohms).
θ	angle to the vertical.
κ_r	Complex relative permittivity of the ground. In numerical examples assumed to be 5-j18 (low conductivity) or 20-j180 (high conductivity).
λ	Wavelength (m).
σ	Conductivity of the ground (mhos/m). In numerical examples assumed to be 10^{-3} mhos/m (low conductivity) or 10^{-2} mhos/m (high conductivity).
ω	Angular frequency (radians/sec). In numerical examples $\omega/2\pi$ is taken to be 1 Mc/s.

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SUMMARY

This report describes a theoretical investigation into earth systems for medium-wave transmitting stations. The change in the groundwave field strength for a given input current, resulting from the installation of an earth system, has been calculated by the compensation-theorem method. Attention is mainly directed to perfectly-conducting circular earth systems and systems of 72 radial wires, but some consideration is also given to semi-circular earth systems. The results have been combined with those given in earlier reports dealing with the input impedance, in order to obtain the effect of the earth system on the power efficiency.

The effect of the earth system on the vertical radiation pattern is not of practical importance, but it has been derived theoretically in order to check the method of analysis by comparison with experimental results.

Suggestions are made for earth systems at future stations.

1. INTRODUCTION.

This report is concerned with the earth-systems associated with grounded aerials at medium-wave broadcasting stations. Three of the properties of a medium-wave aerial will be influenced by the nature and extent of the earth system:

a. The input impedance, the resistive component of which is the more important.

b. The groundwave field strength at a distant point for a given input current.

c. The vertical radiation pattern, which determines the degree to which the night-time service area is restricted by fading.

(a) and (b) are important in so far as together they determine the efficiency, which is proportional to the square of the groundwave field strength for a given input power.

The results of a theoretical investigation into the effect of the ground constants, and of an earth system, on the impedance of a vertical aerial were given in

Research Department Reports* Nos. E.042[1] and E.042/2[2]. Section 2 of this report is concerned with the effect of the presence of the earth system on the groundwave field strength for a given input current. In Section 3 the results of Section 2 are combined with those described in E.042 and E.042/2 to obtain the effect of the earth system on the efficiency of the radiating system.

The method of analysis, which is based on the "compensation theorem" method described in an I.E.E. Monograph[3], is discussed in Section 2.1. It is applied to perfectly-conducting circular earth systems and radial-wire earth systems in Sections 2.2 and 2.3 respectively. Section 2.4 is concerned with perfectly-conducting semicircular earth systems, which are of considerable theoretical interest, since a study of their behaviour assists in the understanding of the results for circular earth systems.

The majority of the results tacitly assume that the current distribution on the aerial corresponds to a sinusoidal standing-wave pattern, but the effect of feed current in an antifading mast-radiator, which is greater than might have been supposed, is considered in Section 2.5.

In Section 3.1 the results obtained in Section 2 are combined with those obtained for the impedance in E.042 and E.042/2 to obtain the effect of the earth system on the efficiency of the aerial. These are the results of greatest practical importance, since the only object of using an extensive earth system is to increase efficiency; it is immaterial whether this result is obtained by reducing the input resistance or increasing the groundwave field strength for a given input current. Attempts have been made to obtain the effect on the efficiency more directly by considering the dissipation of energy in the ground surrounding the aerial; this procedure may be termed the "absorbed-energy method". In Section 3.2, the results obtained by the absorbed-energy method are compared with those of this report, and limitations of the method are discussed. An attempt to check the results of Section 3.1 experimentally is summarised in Section 3.3.

The effect of the earth system on the vertical radiation pattern was at one time thought to be significant. Since imperfect ground conductivity can cause an increase in skywave radiation at certain elevations, it seemed reasonable to believe that a local improvement in the conductivity, due to the installation of an extensive earth system, would improve the anti-fading performance. An experimental investigation using small-scale models[4, 5] had, however, shown that a worth-while improvement cannot be obtained by the use of any earth system that would be practicable in this country. Nevertheless it was considered desirable to determine the effect of the earth system on the vertical radiation pattern theoretically, if only to test the method of analysis by comparison of theoretical and experimental results. This has been done in Section 4.

Rationalised M.K.S. units are used, and the time factor $\exp(j\omega t)$ is suppressed.

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*For the sake of brevity each Research Department Report will be referred to, after the first occasion, by the number only, e.g. "E.042".

2. THE GROUNDWAVE FIELD FOR A GIVEN AERIAL CURRENT.

2.1. Method of Analysis.

The method is based upon the extension of the compensation theorem, which is well-known in connection with electrical networks, to three-dimensional systems. The result, Equation 8, was originally derived from a general formula already published*; this derivation is given in Appendix 7.1. An alternative and simpler approach will be outlined here; this is confined to systems having circular symmetry, and may be open to certain theoretical objections, but since the argument is physical rather than mathematical a clearer understanding of the steps is possible. The procedure is similar to that used in E.042 to obtain the effect of the ground constants, and of the earth system, on the input impedance.

In the first instance attention will be confined to an idealised perfectly-conducting earth system, such as would be formed by a metal cylinder embedded in the ground (E.042, Fig. 1). Commencing with the case of a vertical aerial carrying a given current distribution over uniform imperfectly-conducting ground, the compensation theorem will be used to determine the change in the groundwave field at a distant point caused by the addition of the earth system. It should be noted that when the input impedance was considered in E.042, perfectly-conducting ground (rather than imperfectly-conducting ground) was taken as the starting point, the compensation theorem being used to obtain the effect of imperfect ground conductivity outside the earth system. A similar approach would not be possible when considering the groundwave field†.

Following the procedure adopted in E.042, the electrical properties of the ground will be represented in terms of its surface impedance. (The justification of this is discussed in E.042, Section 2.) The ground will be regarded as a thin film having a surface impedance $\mathfrak{z}$ which is backed by a substance of infinite permeability (Fig. 1(a)). $\mathfrak{z}$ is equal to $\eta/\sqrt{\kappa_r}$ where $\eta = 120\pi$ ohms is the intrinsic impedance of free space and κ_r is the relative permittivity of the ground. Conduction is taken into account by assigning to κ_r an imaginary part equal to $-j60\sigma\lambda$, where σ is the conductivity in mhos/metre and λ is the free-space wavelength in metres. It is required to determine the effect on the groundwave field at some distant point of reducing the surface impedance to zero within a circular area, as shown in Fig. 1(b), while maintaining the current in the aerial unchanged.

Consider two cases:

Case 1 (Fig. 1(a)). The surface impedance is $\mathfrak{z}$. Suppose that a current I_0 is impressed between the aerial terminals, and let this result, at a radius r, in a total surface current I directed radially inwards. Let the groundwave field strength at some distant point be E.

Case 2 (Fig. 1(b)). The surface impedance remains $\mathfrak{z}$ for $r > r_0$, where r_0 is the radius of the earth system, but is reduced to zero for $r < r_0$. As a result let I and E change to I' and E', I_0 being held constant.

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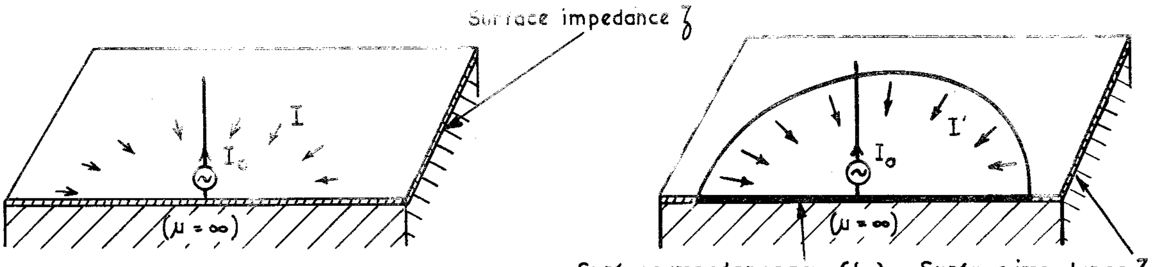
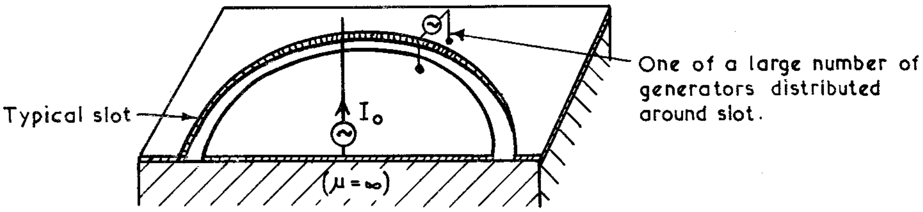
*Reference 3, Equation 7. The application of this result to the present problem was proposed in Section 6.3 of the same paper.

†This would amount to substituting the compensation theorem for Sommerfeld's theory. See second footnote to Appendix 7.4 and Reference 3, Section 6.3.



(a)
Case 1
Groundwave field strength E at some distant point

(b)
Case 2
Groundwave field-strength E′ at the same distant point


Typical slot
I_0
(μ = ∞)
One of a large number of generators distributed around slot.

(c)
Case 2 simulated by energising circular slots.

Fig. 1

It is required to find $(E'-E)/E$, the relative change in groundwave field from Case 1 to Case 2.

According to the compensation theorem, a change from Case 1 to Case 2 could be simulated by cutting a large number of circular slots in the surface film, and maintaining a potential difference across each slot by means of a large number of generators distributed round its circumference, so as to compensate for the opposition of the surface impedance to the flow of surface current. Fig. 1(c) shows a typical slot. Thus the determination of $E'-E$, the change in groundwave field due to a change from Case 1 to Case 2, has been reduced to the calculation of the radiation field of a set of circular slots. The first step is to compare the field of a circular slot with that of a vertical aerial.

Let the groundwave field of a vertical aerial, effective height h, carrying a current I_v, above a perfectly-conducting ground-plane be E_v at some distant point. Let a circular slot, radius r, in the ground plane with a potential difference V_s between its edges result in a groundwave field E_s at the same distance. (V_s is positive when the inner edge of the slot is positive.) It may be shown that

$$\frac{E_s}{E_v} = \frac{V_s}{\eta I_v} \cdot \frac{r}{h} \cdot 2\pi \, jJ_1(\beta r) \tag{1}$$

where $\eta = 120\pi$ ohms is the intrinsic impedance of free space, and $\beta = 2\pi/\lambda$ is the phase constant. The Bessel function in this result occurs also in the formula for the field of a loop; indeed a circular slot is equivalent to a magnetic loop carrying a magnetic current, e.g. a toroidal core with a current-carrying winding laid upon the ground plane. It should be noted that the field of a slot with a given potential difference between its edges in an infinite perfectly-conducting sheet is not affected by conditions at the rear of the sheet.

In order to make use of (1) it is necessary to assume that the replacement of the perfectly-conducting sheet by a sheet of finite surface impedance will have the same effect on the groundwave field of a circular slot as on that of a vertical aerial; i.e. that the attenuation factor is the same for both aerials. This assumption can be justified by the application of the reciprocity principle, but the justification will not be given here, since the alternative approach given in Appendix 1 overcomes this and other theoretical difficulties.

Referring again to Fig. 1, the impedance of an elementary annulus of inner and outer radii r, $r + dr$, is $\mathfrak{z}\,dr/2\pi r$. If one slot is cut in this annulus, the E.M.F. of the generators connected to its edges must be inwardly-directed and equal to $I'(r)\mathfrak{z}dr/2\pi r$ in order to overcome the effect of the impedance of the annulus. Using (1), it is found that the ratio of the field of the slot to that of the vertical aerial is

$$\frac{I'(r)\mathfrak{z}dr}{2\pi r} \cdot \frac{r}{I_0 h \eta} \cdot 2\pi \, jJ_1(\beta r)$$

where h is the effective height of the aerial, i.e. the moment divided by I_0. Summing the contributions from all the slots in the elementary annuli,

$$\frac{E'-E}{E} = \int_0^{r_0} \frac{I'(r)\mathfrak{z}dr}{2\pi r} \cdot \frac{r}{I_0 h \eta} \cdot 2\pi \, jJ_1(\beta r)$$

$$= \frac{j}{I_0 h \eta} \int_0^{r_0} \mathfrak{z} I'(r) J_1(\beta r)\,dr \tag{2}$$

where r_0 is the radius of the earth system.

Equation 2 is applicable only to perfectly-conducting earth systems, which reduce the surface impedance to zero. If, however, the presence of the earth system reduces the surface impedance from $\mathfrak{z}$ to $\mathfrak{z}'$ rather than to zero, $\mathfrak{z}$ in (2) should be replaced by $(\mathfrak{z} - \mathfrak{z}')$. The equation thus becomes

$$\frac{E'-E}{E} = -\frac{j}{I_0 h \eta}\int_0^{r_0} (\mathfrak{z}'-\mathfrak{z}) I'(r) J_1(\beta r)\, dr \qquad (3)$$

This result is derived by an alternative method in Appendix 7·1.

In the case of a radial-wire earth system $\mathfrak{z}'$ is made up of the parallel combination of $\mathfrak{z}$, the surface impedance of the ground, and the surface reactance of the grating formed by the radial wires. This subject was discussed in E.042, and the value of $\mathfrak{z}'$ in particular cases was illustrated graphically. Since $\mathfrak{z}'$ will depend upon the spacing of the wires, which is proportional to r, the distance from the centre, it must be kept within the integral.

$I'(r)$ in Equations 2 and 3 is the total surface current, at a radius r, in the presence of the earth system. This quantity is clearly not known exactly; if the fields were known in the presence of the earth system this analysis would be unnecessary. Since, however, Equation 2 or 3 is an expression for the change in the groundwave field due to the presence of the earth system, and since this change will be comparatively small, some degree of approximation is permissible. $I'(r)$ will therefore be replaced by $I_1(r)$, the total surface current that would exist if the ground were everywhere perfectly-conducting. When the earth system is perfectly-conducting, $I'(r)$ will differ from $I_1(r)$ only because of reflection at the boundary; it is considered that the resulting error in $(E'-E)$ will be small in this case. In the case of an earth system that is far from perfectly-conducting, $I'(r)$ will be reduced in amplitude and retarded in phase owing to groundwave attenuation, and the contributions to $(E'-E)$ from the outer regions of the earth system will therefore be less accurately assessed. Nevertheless these contributions will be smaller, the outer regions of the earth system being less effective, and the resulting errors will again be small.

It is difficult to arrive at a quantitative estimate of the error caused by the replacement of $I'(r)$ by $I_1(r)$ in (2) and (3), but it seems reasonable to believe that the presence of an earth system will tend to made the total surface current approach $I_1(r)$, the value for perfectly-conducting ground. In other words, the difference between $I'(r)$ and $I_1(r)$ should be less than that between $I(r)$ and $I_1(r)$, a quantity which may be calculated. This approach leads to the conclusion that the approximation is sufficiently accurate for engineering purposes, provided that the radius of the earth system does not exceed one wavelength.

Making the approximation, (3) becomes

$$\frac{E'-E}{E} \simeq -\frac{j}{I_0 h \eta}\int_0^{r_0} (\mathfrak{z}'-\mathfrak{z}) I_1(r) J_1(\beta r)\, dr \qquad (4)$$

A formula for $I_1(r)$, derived by G.H. Brown[6], is reproduced in E.042, together with a graphical representation of the result in particular cases.

The integral in Equation 4 requires to be evaluated numerically, even in the case of a perfectly-conducting earth system, when $\mathfrak{z}'$ vanishes and $\mathfrak{z}$ can be taken outside. A labour-saving method is described in Appendix 7.2.

2.2. Perfectly-conducting Circular Earth Systems.

The presence of the earth system affects both the magnitude and the phase of the groundwave field for a given current. $(E'-E)$, the change in field strength caused by the earth system, is proportional to the surface impedance of the ground, which is inversely proportional to $\sqrt{\kappa_r}$, the square root of the complex relative permittivity. It is therefore convenient to present the results in a form which allows their application to any ground constants by dividing by $\sqrt{\kappa_r}$.

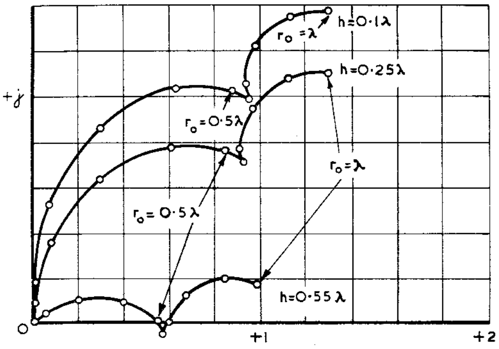


Fig. 2 - $\sqrt{\kappa_r}(E'-E)/E$ for a perfect circular earth system.
Points plotted at 0·1λ intervals of r_0.

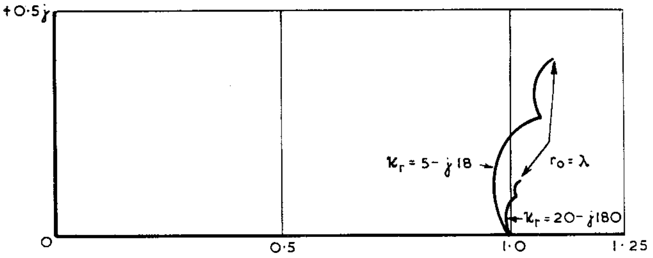


Fig. 3 - E'/E for a perfect circular earth system.
Aerial height 0·25λ.

Fig. 2 shows $\sqrt{\kappa_r}(E'-E)/E$ plotted on the Argand diagram as a function of the radius of the earth system in wavelengths, for three aerial heights, 0·1λ, 0·25λ and 0·55λ. In order to obtain $(E'-E)/E$, the diagram should be scaled in proportion to $1/\sqrt{|\kappa_r|}$ and rotated through an angle $\{-\frac{1}{2}$ phase $(\kappa_r)\}$. E'/E, the complex ratio of the field strength in the presence of the earth system to that in its absence, is then obtained by moving the origin to the point $(-1+j0)$. As an example Fig. 3 shows E'/E for a quarter-wave aerial.

For practical purposes it is the magnitude of E', rather than its phase, that is important. Figs. 5(a), 6(a) and 7(a) show $|E'/E|$ for aerials of height 0·1 λ, 0·25λ and 0·55λ respectively; in each case two types of ground are considered: highly-conducting ground corresponding to a good B.B.C. site, (σ = 10^{-2} mhos/m, κ_r = 20-j180 at 1 Mc/s), and ground of the lowest conductivity of practical importance found in this country (σ = 10^{-3} mhos/m, κ_r = 5-j18 at 1 Mc/s).

The most striking feature of the results is the fact that the groundwave field strength does not increase steadily as the radius of the earth system is increased. When E'/E is plotted on the Argand diagram, it traces out a locus resembling a cycloid, becoming stationary at each cusp. As a result the magnitude of

E'/E increases at a non-uniform rate, sometimes decreasing slightly for an increase in the radius of the earth system. The cusps in Fig. 2, which correspond to turning points in Figs. 5-7, indicate that E', considered as a function of r_0, is stationary for certain values of r_0. In other words a small change in the radius of the earth system from r_0 to $r_0 + dr_0$ would have no first-order effect on the magnitude or phase of the groundwave field. These values of r_0 occur when $J_1(\beta r_0) = 0$. A loop of this radius would not radiate in its own plane. This fact, though of some interest, does not offer a very satisfactory physical explanation of the phenomenon.

The cycloidal form of the curves of Fig. 2 suggested that $E'-E$, the change in groundwave field caused by the earth system, is the sum of two components, one increasing steadily as the radius of the earth system increases, the other falling off slowly in magnitude while its phase becomes progressively more retarded. The first component was thought to be associated with the elimination of groundwave attenuation along that part of the path of propagation passing over the earth system. The second component, which for earth systems of large radius is retarded by one cycle for an increase of one half-wavelength in radius, was thought to be due to reflection from the discontinuity in conductivity occurring at the boundary of the earth system. This explanation has been confirmed by considering a semicircular earth system, which could be oriented so as to produce either component of $(E'-E)$ separately. The results of this investigation are given in Section 2.4.

In what follows the general trend of the curves of Fig. 2 will be considered, the cusps caused by reflection at the boundary being disregarded. The behaviour of E'/E as a function of the radius of the earth system is found to be connected with the way in which the groundwave attenuation factor varies with distance at short distances. As might be expected, the earth system tends to overcome the attenuation and phase retardation caused by finite ground conductivity.

The effect of the earth system on the groundwave field depends appreciably on the height of the aerial. Comparing a $0{\cdot}55\lambda$ aerial with a quarter-wave aerial, it is found that increase in the magnitude of the groundwave field due to the earth system is greater in the former case, while the advancement of the phase is greater in the latter case. This effect is related to the phenomenon of height-gain[7], which is caused by the imperfect conductivity of the ground in the neighbourhood of the aerial. In the absence of an extensive earth system the height-gain factor of a medium-wave aerial decreases in magnitude and advances in phase as the height is increased. The earth system, by effectively improving the conductivity locally, tends to offset height-gain. This it does by increasing the magnitude of the field more when the aerial is high than when it is low, but advancing the phase of the field of a low aerial more than that of a high aerial. Calculation has shown that a perfectly-conducting earth system would require to be several wavelengths in radius in order to offset height-gain substantially.

2.3. Radial-wire Earth Systems.

Since the surface impedance of the ground in the presence of the earth system varies with the distance from the centre, the numerical integration must be performed for each value of the ground constants considered. For this reason it is not possible to present the results in the form of universal curves applicable to any ground constants, as was done in the case of perfectly-conducting earth systems (Fig. 2). Instead $(E'-E)/E$ is plotted in Fig. 4 for the two values of the ground constants previously considered. These are

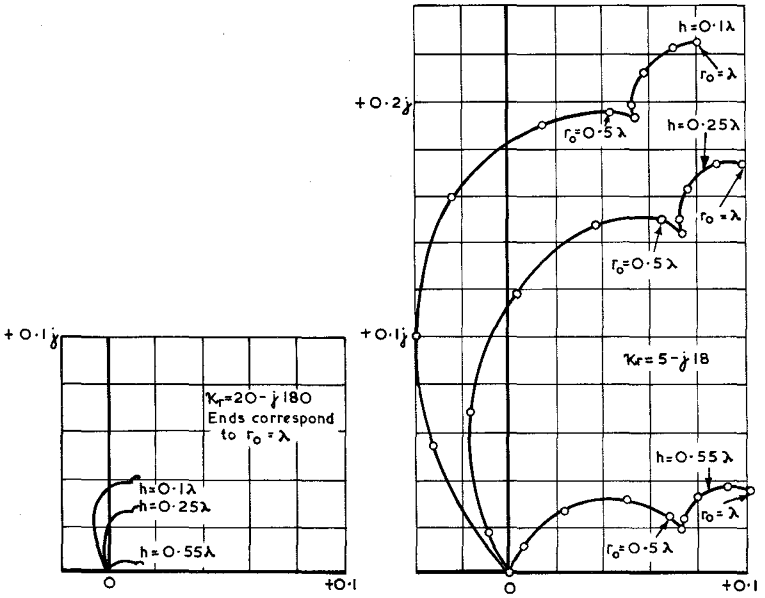


Fig. 4 - $(E'-E)/E$ for a radial-wire earth system (72 wires).

Points plotted at 0·1 λ intervals of r_0.

$$K_r = 20-j180 \quad (\sigma = 10^{-2} \text{ mhos/m for 1 Mc/s})$$

and

$$K_r = 5-j18 \quad (\sigma = 10^{-3} \text{ mhos/m for 1 Mc/s})$$

$|E'/E|$ is shown in Figs. 5(b), 6(b), 7(b).

The results show that the innermost portions of the earth system have an effect similar to that of a perfectly-conducting earth system, but that the outer portions (i.e. $r/\lambda > 0{\cdot}5$) have little effect, as might be expected.

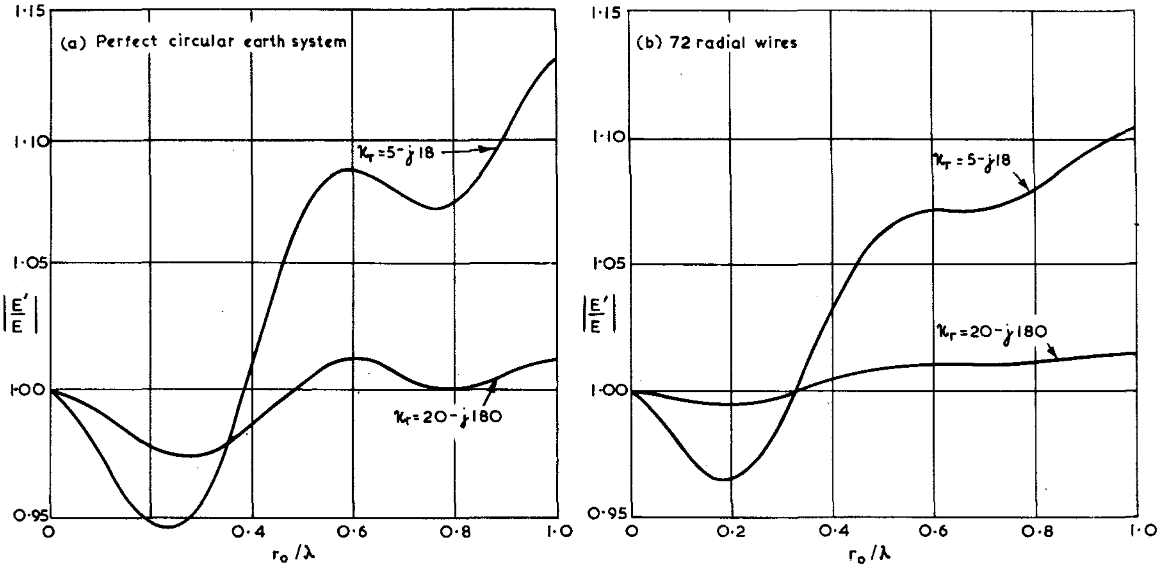


Fig. 5 - $\left|\frac{E'}{E}\right|$ for a 0·1λ aerial.

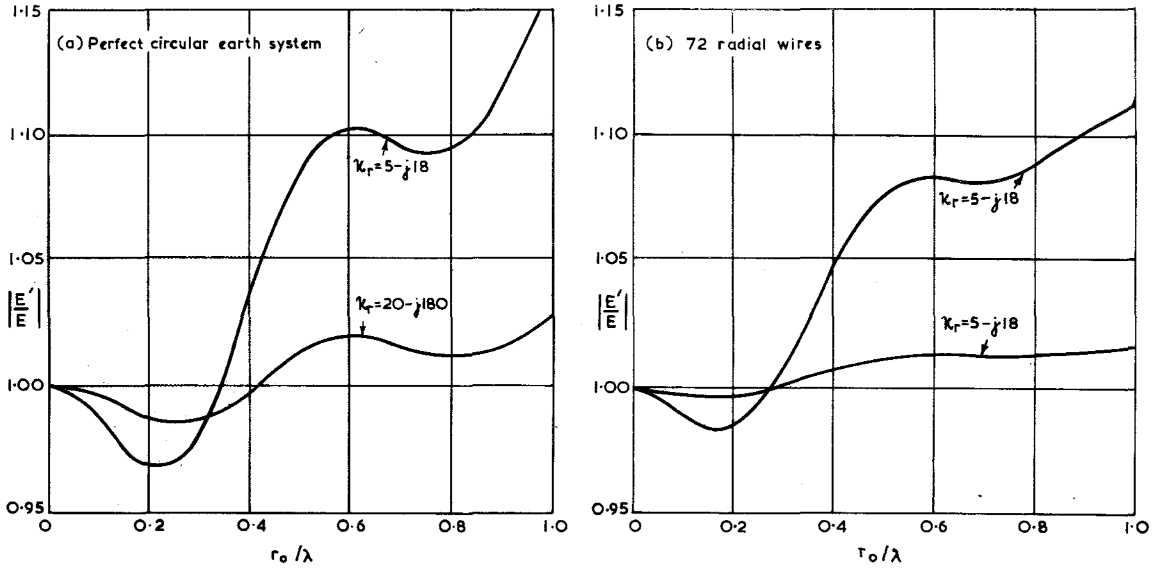


Fig. 6 - $\left|\frac{E'}{E}\right|$ for a 0·25λ aerial.

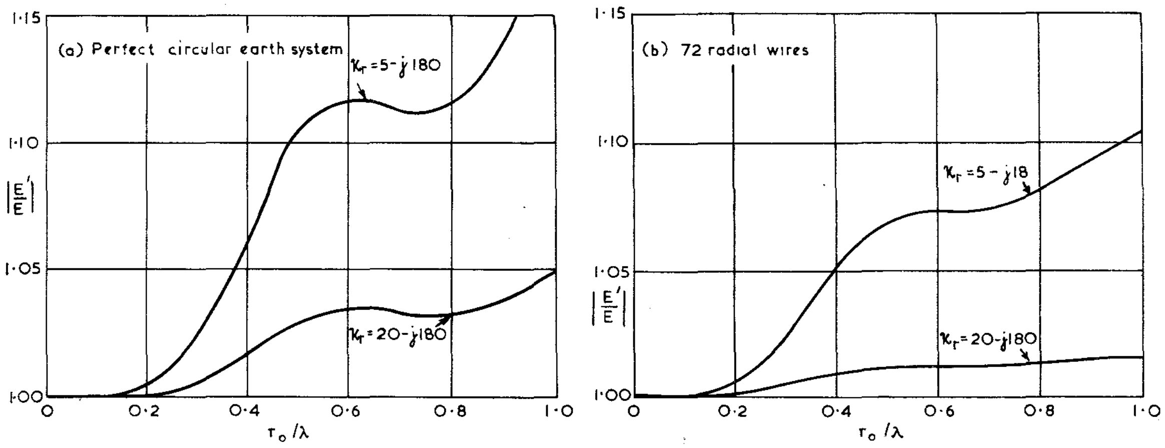


Fig. 7 - $\left|\frac{E'}{E}\right|$ for a $0{\cdot}55\lambda$ aerial.

2.4. Perfectly-conducting Semicircular Earth Systems.

There are three reasons for considering semicircular earth systems. In the first place it was desired to test a possible explanation, discussed in Section 2.2, for the cycloidal shape of the curves of Fig. 2. Secondly, it was hoped that an experiment with a semicircular earth system would provide a convenient means of checking the method of analysis. Finally, it had been suggested that on a site of limited area it might be desirable to favour particular directions by means of an asymmetrical earth system.

A serious difficulty is encountered when applying the compensation theorem method to an exactly semicircular earth system, as shown in Fig. 8(a). It is

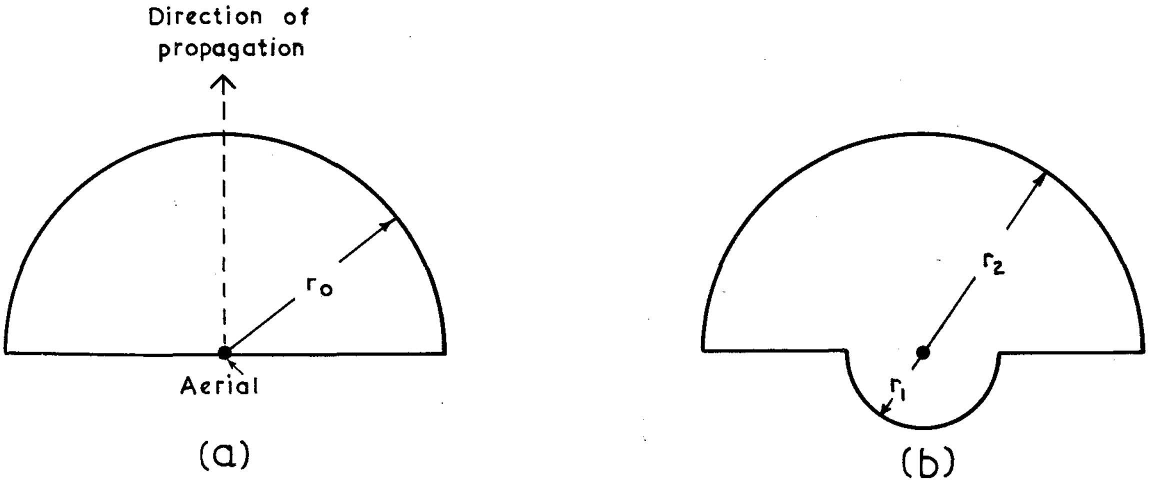


Fig. 8 - Semicircular earth systems.

necessary to assume that the flow of surface current over the earth system is substantially the same as it would be if the ground were everywhere perfectly-conducting. This assumption cannot be true in the immediate vicinity of the aerial, since an infinite impedance would be presented by the ground to current diverging from the base of the aerial away from the earth system. Clearly all the surface current must flow initially in the earth system at very small radii, but must eventually spread out into the ground from the straight edge of the earth system at larger radii.

The difficulty referred to above may be avoided by considering an earth system of the form shown in Fig. 8(b). This consists of two semicircular earth systems placed back to back. Provided that r_1, the smaller radius, is not too small the compensation theorem may be used with reasonable accuracy. In this connection it should be emphasised that a first-order error in assessing the surface current density will cause a first-order error in $E'-E$, the *change* in groundwave field due to the earth system, not in E'. If $E'-E$ is itself fairly small, a small error in it may be tolerated.

It is thought that reasonable accuracy will be obtained provided that r_1, the smaller radius, is at least $0{\cdot}05\lambda$.

The procedure to be adopted is to begin by ignoring the difficulty discussed above, and to compute the effect of the completely semicircular earth system of Fig. 8(a). The results may then be applied to the more realistic earth system of Fig. 8(b) by adding the contributions to the field from the two semicircular earth systems of differing radii oriented in opposite directions. The initial consideration of Fig. 8(a) is simply a convenient method of performing in two parts the integration required to obtain the effect of the earth system of Fig. 8(b).

Equation 4, Section 2.1, was an expression in the form of an integral for the relative change in the groundwave field-strength due to the addition of a circular earth system, assuming the aerial current to be held constant. It is shown in Appendix 7.3 that (4) should be modified for the semicircular earth system of Fig. 8(a) by replacing the Bessel function $J\,(\beta r)$ by the function $F(\beta r)$, defined by

$$F(x) \quad = \quad \tfrac{1}{2}J_1(x) - \frac{j}{\pi}\left\{ J_0(x) - 2\sum_{1}^{\infty} \frac{J_{2m}(x)}{4m^2-1} \right\} \tag{5}$$

Then

$$\frac{E'-E}{E} \quad = -\frac{j}{I_0 h\eta}\int_0^{r_0} (\mathfrak{z}'-\mathfrak{z}) I_1(r) F(\beta r)\,dr \tag{6}$$

If the earth system of Fig. 8(a) is rotated through 180°, so that the initial portion of the path of propagation no longer passes over it, the sign of the imaginary part of $F(x)$ should be reversed. (This fact may be checked by observing that two semicircular earth systems of the same radius oriented in opposite directions are equivalent to one circular earth system.)

Calculations have been carried out only for perfectly-conducting earth systems. The integration was performed graphically. Fig. 9 shows the results for aerials 0·1λ and 0·55λ high, presented in the same form as the results for circular earth systems shown in Fig. 2. In order to apply these results to the composite earth system of Fig. 8(b), the complex values of $\sqrt{K_r}(E'-E)/E$ for the two semicircular portions, radii r_1 and r_2, should be read from the curve and added. One of these, corresponding to the portion extending in the forward direction, should be read from the continuous curve, the other from the broken curve.

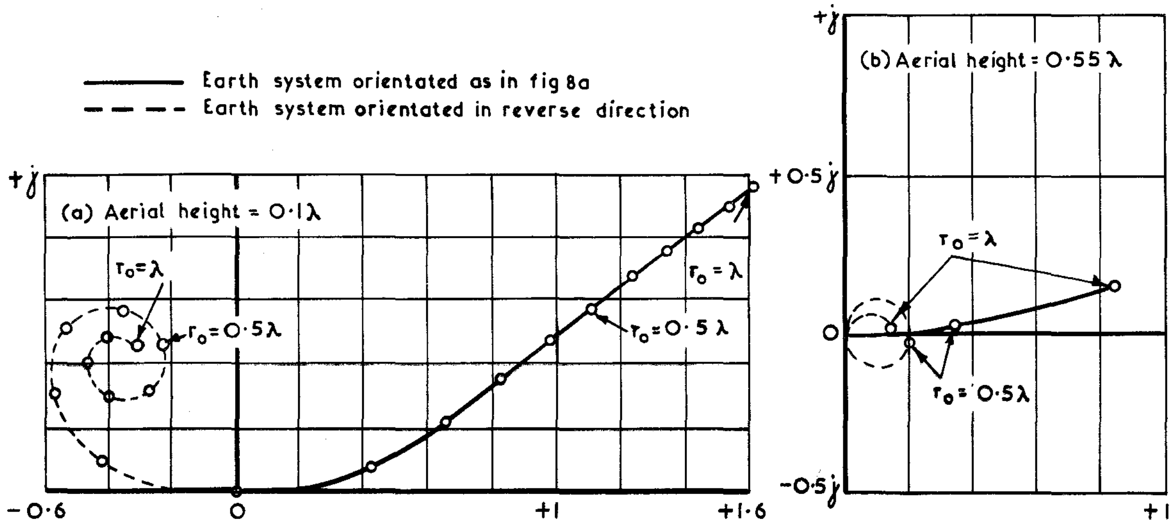


Fig. 9 - $\sqrt{K_r}(E'-E)/E$ for a perfect semicircular earth system.

The results shown in Fig. 9 confirm the tentative explanation given in Section 2.2 for the cycloidal form of the curves of Fig. 2. It is seen that the contribution to the total field caused by the forward half of a circular earth system increases steadily in magnitude with the radius. The spiral form of the broken curves, corresponding to the backward half of a circular earth system, confirms the occurrence of reflection at the boundary.

In order to check the compensation theorem method of analysis experimentally, measurements were made to determine the change in the groundwave field of a vertical aerial caused by rotating a large semicircular earth system from the forward to the backward position. Since the impedance of the aerial would not be changed, and since the rotation could be carried out rapidly, this experiment was thought to provide a very sensitive test. The equipment, which had been set up for an investigation into the effect of earth systems on the vertical radiation pattern, has been described in Research Department Reports Nos. E.041[4] and E.043[5]. The results to be given below were mentioned in the last-named report.

The measurements were made at a frequency of 500 Mc/s, and the ground was simulated by a 0·6% salt water solution, which had a complex relative permittivity of 80-j36 at the frequency of operation. The large real part of the permittivity would not be realised in practice, but had the advantage of causing a rapid initial rate of groundwave attenuation. Hence the effect of an earth system on the groundwave field was greater than for a typical broadcasting site.

The earth system was of the type shown in Fig. 8(b), r_1 and r_2 being $0{\cdot}1\lambda$ and $0{\cdot}5\lambda$ respectively. It was made up of a cylindrical box $0{\cdot}1\lambda$ in radius, which formed a mounting for the aerial and a container for the matching circuits, and a semicircular sheet $0{\cdot}5\lambda$ in radius. The latter could be screwed to the cylindrical box in either of two positions, without causing any other change to the system.

Two heights of aerial were used, $0{\cdot}55\lambda$ and $0{\cdot}1\lambda$. As an example, the effect of the orientation of the earth system on the groundwave field of the $0{\cdot}1\lambda$ aerial will be derived from the results shown in Fig. 9.

$\sqrt{K_r}(E'-E)/E$ for the component semicircles is as follows:

Position a.

$0{\cdot}5\lambda$ semicircle in forward direction	$1{\cdot}11 + j0{\cdot}57$
$0{\cdot}1\lambda$ semicircle in backword direction	$-0{\cdot}42 + j0{\cdot}10$
Total	$0{\cdot}69 + j0{\cdot}67$
	$= 0{\cdot}96\underline{/44^{\circ}}$

Position b.

$0{\cdot}5\lambda$ semicircle in backward direction	$-0{\cdot}23 + j0{\cdot}46$
$0{\cdot}1\lambda$ semicircle in forward direction	$0{\cdot}42 + j0{\cdot}08$
Total	$0{\cdot}19 + j0{\cdot}54$
	$(= 0{\cdot}57\underline{/70{\cdot}5^{\circ}})$

Now $$\sqrt{K_r} = \sqrt{80-j36} = 9{\cdot}35\underline{/-12^{\circ}}.$$

Dividing by this we find that $(E'-E)/E$ is $0{\cdot}103\underline{/56^{\circ}}$ for position a and $0{\cdot}061\underline{/82{\cdot}5^{\circ}}$ for position b.

Adding unity, we find that E'/E is equal to $1{\cdot}061\underline{/4{\cdot}4^{\circ}}$ for position a and $1{\cdot}010\underline{/3{\cdot}4^{\circ}}$ for position b.

The ratio of the magnitudes is $1{\cdot}061/1{\cdot}010 = 1{\cdot}050$. Thus the composite earth system shows a slight directional tendency, the groundwave field in the direction in which the earth system is extended to $0{\cdot}5\lambda$ being 5% greater than in the opposite direction.

In the case of the $0{\cdot}55\lambda$ aerial, the contribution from the $0{\cdot}1\lambda$ semicircle is found to be almost negligible. The ratio of the forward to backward field strength is found to be 1·015.

Experimental and theoretical forward/backward ratios are compared below.

	$\underline{0{\cdot}1\lambda \text{ aerial}}$	$\underline{0{\cdot}55\lambda \text{ aerial}}$
Theoretical	1·05	1·015
Measured	1·07	1·00

It is possible that errors of 2% could occur in the measurements, and the result is therefore inconclusive. Nevertheless the result of most practical importance, that extending the earth system in a preferred direction has very little effect on the groundwave field, is confirmed by the experiment.

2.5. The Effect of Feed Current.

Up to this point the current distribution upon a vertical aerial has been assumed to correspond to a sinusoidal standing-wave pattern with a velocity of propagation along the aerial equal to the free-space velocity. This assumption is substantially true for short aerials not much more than one quarter wavelength high, but is not true for an antifading mast-radiator, whose height is approximately $0{\cdot}5\lambda$. In the latter case the current distribution differs from the idealised sinusoidal form in two respects[8].

In the first place, the velocity of propagation along the aerial is less than that of light, but by estimating the effect of the reduced velocity on the total surface current it is found that the results given above will not be affected greatly. The results computed for an aerial of height $0{\cdot}55\lambda$ will in practice be more nearly applicable to an aerial whose physical height is $0{\cdot}525\lambda$ and whose vertical radiation pattern indicates an apparent height of $0{\cdot}55\lambda$.

Secondly, the current distribution on a base-fed aerial is not co-phased. It is convenient to resolve the total current into two components: the primary current, which corresponds to a sinusoidal standing-wave pattern, and a component in phase-quadrature termed the feed current. The feed current tends to be concentrated more towards the base of the aerial than the primary current; its phase is advanced 90° relative to that of the primary current at the loop.

An adequate approximation to the feed-current distribution is to treat it as the same as the primary current distribution on a quarter-wave aerial. Thus it is possible to take feed-current into account by combining the results given above for $0{\cdot}55\lambda$ and $0{\cdot}25\lambda$ aerials, the fields of the two components being added as vectors. As an example, Fig. 10 shows the effect of a perfectly-conducting circular earth system on the magnitude of the groundwave field of a $0{\cdot}55\lambda$ aerial in three cases. The complex relative permittivity of the ground has been taken as 5-j18, corresponding to a very poor site. Curve 1 is applicable to a typical base-fed mast radiator. Curve 2, taken from Fig. 7, applies to a loop-fed aerial[9], where feed-current is eliminated. Curve 3 applies to a doubly-fed aerial[9] upon which the feed current has been reversed, one unit of power being drawn off at the base while two units are injected at the loop. In computing both curves 1 and 3, the moment of the feed-current distribution has been taken as $0{\cdot}2$ times that of the primary-current distribution.

It will be seen that the increase in groundwave field due to the earth system is greater if the aerial is loop-fed than if it is base-fed, and greater still in the case of a doubly-fed aerial. Now the height-gain factor[7] for a loop-fed aerial is less than that for a base-fed aerial of the same height, and that of a doubly-fed aerial is lower still. The difference between the curves of Fig. 10 is

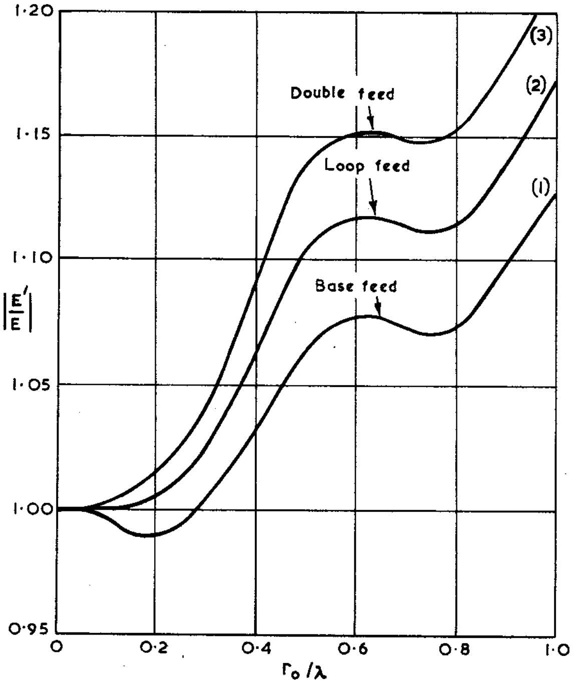


Fig. 10 - $\left|\frac{E'}{E}\right|$ for a 0·55λ aerial with a perfect earth system, radius r_0, over poorly-conducting ground. (κ_r = 5-j18)

consistent with the statement in Section 2.2 that the earth system tends to offset the effect of height-gain.

3. EFFICIENCY.

3.1. From the Impedance and the Groundwave Field.

In E.045 the efficiency of a grounded aerial was defined as follows: Let the base of the aerial be at A, and let B be a distant point on the ground. Then the efficiency is given by

$$\gamma = \left[\frac{\text{(Field at B for unit power)}}{\begin{pmatrix}\text{Theoretical field at B for unit power}\\ \text{assuming perfectly-conducting ground}\end{pmatrix} \times F_{AB}}\right]^2 \qquad (7)$$

where F_{AB}, the attenuation factor for the path AB*, is defined by supposing a very short grounded aerial to be placed at A, and writing

$$F_{AB} = \frac{\text{(Field at B for unit input current)}}{\begin{pmatrix}\text{Theoretical field at B for unit input}\\ \text{current assuming a perfectly-conducting ground}\end{pmatrix}} \qquad (8)$$

---

*This was denoted by f_{AB} in E.045. The capital has been substituted to avoid confusion with the height-gain factor.

It should be noted that F_{AB} is defined in terms of a hypothetical experiment with a very short aerial. If the experiment were performed with an aerial of appreciable height the right-hand side of Equation 8 would be reduced in proportion to the height-gain factor, which is less than unity for practical medium-wave aerials. The height-gain factor is in fact a measure of the additional groundwave attenuation caused by the finite height of the aerial.

The efficiency may be calculated from

$$\gamma = |f|^2 \left|\frac{E'}{E}\right|^2 \frac{R}{R'} \tag{9}$$

where $|f|$ is the height-gain factor, obtained from E.045.

$|E'/E|$ is the quantity plotted in Figs. 5, 6, 7 and 10, of this report.

R and R' are respectively the resistance of the aerial over perfectly-conducting ground and that over imperfectly-conducting ground in the presence of the earth system. $(R'-R)/R$ is plotted in Figs. 16-22 of E.042/2.

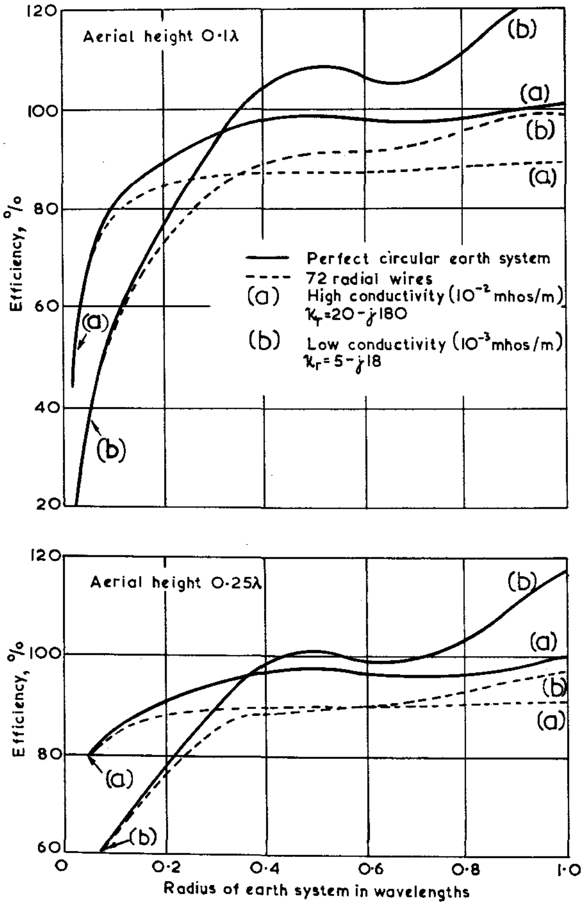


Fig. 11 - The efficiency of short aerials.

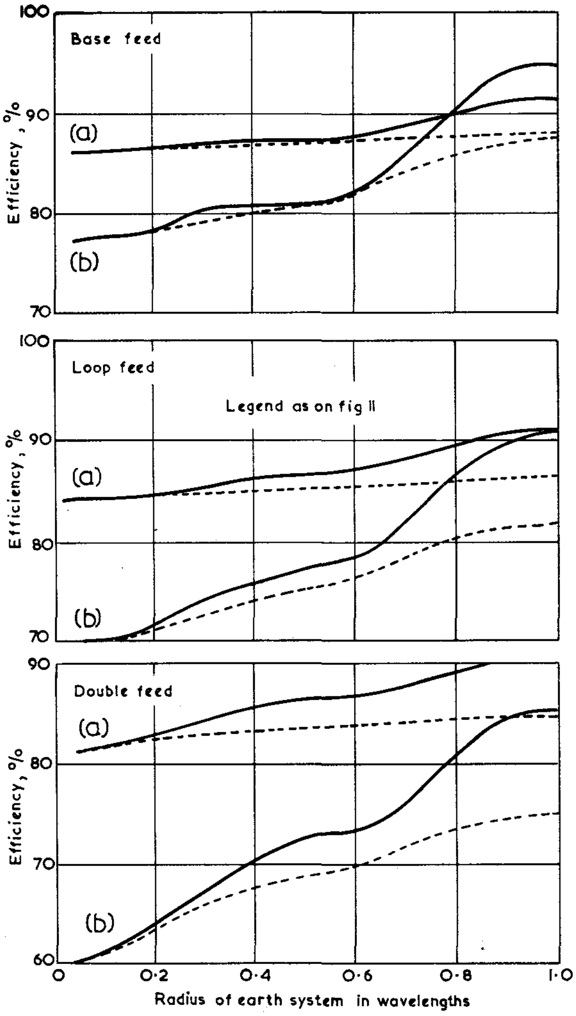


Fig. 12 - The efficiency of a 0·55λ anti-fading mast-radiator.

In Fig. 11 the efficiency obtained from (9) is shown as a function of the radius of the earth system for a $0{\cdot}1\lambda$ aerial and a $0{\cdot}25\lambda$ aerial. Fig. 12 shows the efficiency of an antifading mast-radiator for three methods of feeding: base-feeding, loop-feeding and double-feeding. In taking account of feed current, its effect on the input impedance was calculated by the method outlined in E.042/2, rather than the less accurate method originally proposed in E.042.

In certain cases the efficiency as here defined, exceeds 100%. It would not have been practicable to define efficiency so as to make this impossible, since if the ground is imperfectly-conducting the field at a sufficiently distant point can be increased almost without limit by extending the earth system. In principle, a limit could only be reached by copper-plating the entire globe.

3.2. The Absorbed-energy Method.

In the absence of a more complete theoretical treatment, such as that contained in this report, G.H. Brown,[10,11] and later Abbott[12], have assessed the efficiency of earth systems in terms of the extent to which they reduce the absorption of energy. It was assumed that if an improvement in the earth system reduces the absorption of energy in the region covered by it by 1 kW, the resulting increase in field strength would correspond to an increase of 1 kW in the transmitter power. Even if this assumption is granted, the absorbed-energy approach cannot lead to an estimate of the efficiency, as defined in Section 3.1, but it does enable the calculation of the *change* in efficiency resulting from a given change in the earth system. The absorbed-energy method can be justified by the conservation of energy provided that:

a. the earth system has no effect on the vertical radiation pattern and

b. the earth system does not affect the way in which the surface current varies with distance outside the earth system (since the dissipation of energy in the ground depends on the surface current density).

(b) cannot be checked easily, but it is thought likely that this condition will be satisfied if (a) is satisfied. Experiments [4,5] have shown that (a) is satisfied for earth systems less than one half-wavelength in radius. In view of the fact that the use of the absorbed-energy method, as opposed to the conpensation-theorem method, would reduce the labour of computation in certain problems, comparison of the two methods is desirable.

The method of calculating the absorption of energy has been discussed in Reference 3 Section 6.1, and Reference 1 Section 7.3, and need only be summarised briefly here. The power dW absorbed in an elementary annulus of ground having inner and outer radii r, $r + dr$ is given by

$$dW = (1/4\pi r)\,|I(r)|^2\,\Re(\mathfrak{z})\,dr \qquad (10)$$

where $I(r)$ is the total surface current at radius r and $\Re(\mathfrak{z})$ is the surface resistance. Equation 10 corresponds to G.H. Brown's[10] method, if the real part of the permittivity is neglected in computing $\mathfrak{z}$.

In order to calculate the energy absorbed in an annulus covered by a radial-wire earth system Brown made use of a formula whose accuracy is suspect. The correct procedure is believed to be to replace $\mathfrak{z}$ in (10) by $\mathfrak{z}'$, the surface impedance of the ground and the earth system in parallel. This method was proposed in Reference 3, and has also been put forward independently by Abbott[12].

Fig. 13 shows the variation of the efficiency of $0{\cdot}25\lambda$ and $0{\cdot}55\lambda$ aerials with the radius of a perfectly-conducting earth system. The efficiency has been computed by three methods:

a. By the method of Section 3.1. The compensation theorem has been used to determine the effect of the earth system on the impedance and on the groundwave field for a given current.

b. By ignoring the effect of the earth system on the groundwave field for a given current, and considering only its effect on the input impedance.

c. By the absorbed-energy method. Since this method gives only relative rather than absolute efficiences, the curves have been scaled for the best fit with curves (a) at small radii.

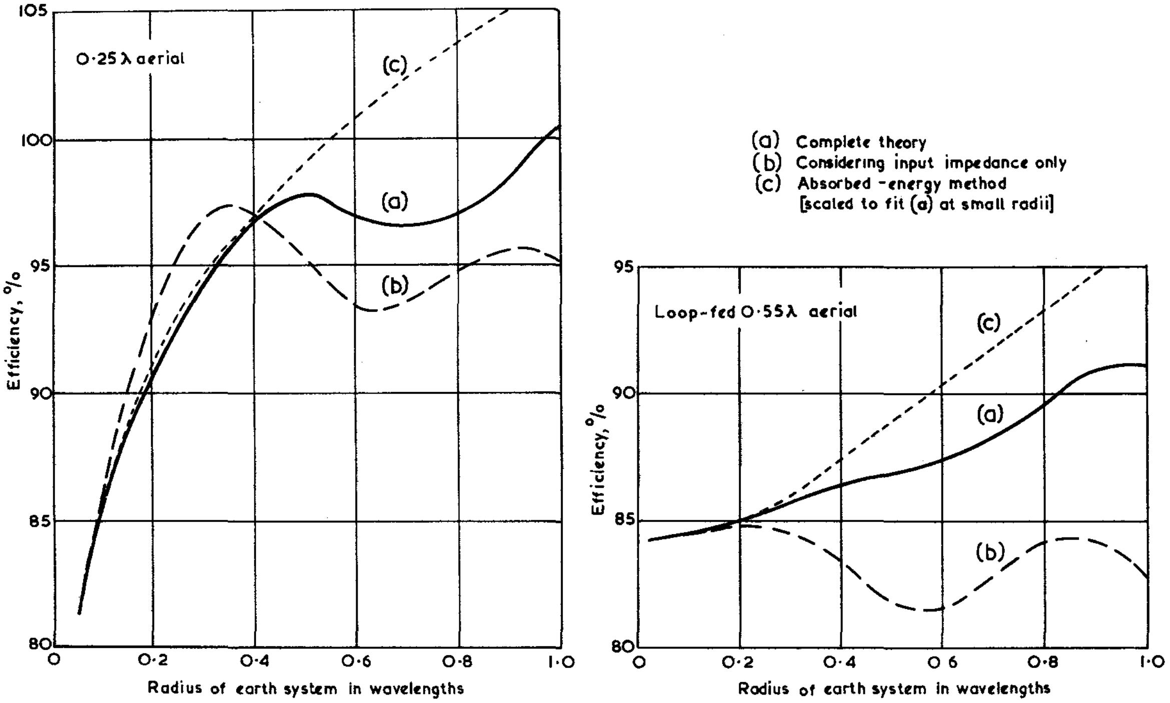


Fig. 13 - Efficiency computed by three methods.
High conductivity (κ_r = 20-j180). Perfectly-conducting earth system.

It will be observed that methods (a) and (b) agree to 1% for radii up to $0{\cdot}125\lambda$ and that (a) and (c) agree to 1% for radii up to $0{\cdot}4\lambda$. Since, however, practical earth systems have far less effect near their outer edges than have perfectly-conducting earth systems, it is considered that the absorbed-energy method can safely be employed for radii up to $0{\cdot}5\lambda$ at least, i.e. to all earth systems at present used in the B.B.C.

Should further calculations become necessary, it would be best to begin by using the compensation-theorem method to calculate the efficiency for one earth system. This should preferably be a perfectly-conducting earth system of small radius—say

$0{\cdot}05\lambda$ — so that its effect on the groundwave field for a given current can be neglected. Having obtained the absolute efficiency in this way for one earth system, the efficiency for other earth-systems could be derived by using the absorbed-energy method to compute differences in efficiency.

Abbott[12] used the absorbed-energy method to deduce the number and length of the radial earth-wires that would be desirable on economic grounds, balancing the cost of buried copper against that of r.f. power. His approach appears to lead to earth systems more than one wavelength in radius in some practical cases, but, owing to the failure of the method at large radii, these results are considered to be invalid. Even if Abbott's economic assumptions are accepted, his method will result in an over-estimate of the optimum radius, but it might also be remarked that different economic considerations would apply in this country. Abbott neglected the cost of land, but in Great Britain land is far more valuable than buried copper; indeed, its value cannot be assessed in terms of money alone.

3.3. Experimental.

Brown, Lewis, and Epstein[11] made measurements at 3 Mc/s to determine the effect of the number and length of radial earth-wires on the efficiency of a short aerial. Unfortunately they made no attempt to assess the ground constants, and the value of their results is therefore limited.

In the B.B.C. advantage was taken of the construction of the new low-power transmitting station at Barnstaple to instal the earth system in three stages. The largest earth system comprised 72 radial wires 0·2 wavelengths long. Measurements of input impedance and efficiency were made at each stage, and the results were compared with theoretical estimates. This work is described in B.B.C. Research Department Technical Memorandum No. E.1028[13].

The results showed approximately the expected changes in power efficiency, but in other respects were in conflict with the theory. In the first place the groundwave field strength for a given input current appeared to increase by 5% between the initial and final stages, although no appreciable change was expected. This discrepancy could have been accounted for by experimental error. On the other hand, the reduction in input resistance was only 9%, whereas a reduction of 19% had been expected. This discrepancy was too large to be accounted for by errors of measurement. It is just possible that the expected change was partly compensated by an increase in loss from some other unknown cause. Although the results of the experiment were inconclusive, they confirmed the fact that there had been a tendency in the B.B.C. to instal more extensive earth systems than was necessary. It would be desirable to repeat the experiment should a favourable opportunity occur. The site should be flat and in an area where the ground conductivity is known to be uniform. The accuracy of measuring small changes in field strength should be improved, possibly by using fixed equipment installed in a building. It would be advisable to check the input impedance at intervals over a period, both before and after the experiment, in order to detect any changes that are not due to the extension of the earth system.

4. THE VERTICAL RADIATION PATTERN.

This section will be concerned mainly with antifading mast radiators having an effective height slightly greater than $0{\cdot}5\lambda$. The vertical radiation pattern exhibits a minimum at an angle to the vertical depending upon the height. The imperfect conductivity of the ground affects the field in the neighbourhood of this minimum in two ways: the minimum is moved to a greater angle from the vertical, and the minimum field-strength is increased. The latter effect is undesirable, and it is of interest to know to what extent it is offset by the installation of an extensive earth system. Only circular perfectly-conducting earth systems will be considered.

Before this problem had been approached theoretically, an experimental investigation had been carried out by means of small-scale models[4, 5]. The results showed that a useful improvement could be effected only in the case of poorly-conducting ground, and then only by employing an earth system at least one wavelength in radius. Such extensive earth systems would hardly be practicable in the B.B.C., and in any case it happens in this country that where the conductivity is low the site and its neighbourhood are far from flat. Site irregularities degrade the vertical radiation pattern to such an extent that the relatively small improvement effected by a large earth system would not be significant.

In spite of the fact that the effect of the earth system on the vertical radiation pattern was known to be unimportant, theoretical calculations have been carried out in order to check the compensation theorem in its application to earth systems by comparing theory with the results of experiment.

The method of analysis described in Section 2.1 can be extended so as to obtain the effect of the earth system on the radiation pattern. The principal modification is the replacement of $J_1(\beta r)$ by $J_1(\beta r \sin \theta)$, where θ is the angle to the vertical. This change is almost self-evident since $J_1(\beta r \sin \theta)$ corresponds to the vertical radiation pattern of a horizontal loop of radius r. Some computations were carried out in this way, but the method was abandoned owing to the labour involved, since the graphical integration had to be performed afresh for each value of θ considered. Instead an alternative approach was adopted, taking the radiation pattern over perfectly-conducting ground as a starting point, and deriving the effect of the imperfect conductivity of the ground outside the earth system by the compensation theorem method. By restricting consideration to perfectly-conducting earth systems at least one wavelength in radius, it was possible to obtain a solution in terms of tabulated functions with no appreciable sacrifice of accuracy. (This approach could not have been applied to the groundwave field, since the approximations would have failed and the integral would have diverged.)

Such comparisons as have been made between the two methods—i.e. taking perfectly-conducting and imperfectly-conducting ground respectively as the starting point—have shown fairly good agreement.

Suppose that over perfectly-conducting ground a current I_0 impressed between the aerial terminals results in a field-strength $E_1(\theta)$ at an angle θ to the vertical, and at some great distance which will be regarded as fixed. Now suppose that the surface impedance of the ground is changed from zero to $\mathfrak{z}$ at all points outside a circular region of radius r, and that as a result $E_1(\theta)$ is changed to $E'(\theta)$.

It is shown in Appendix 7.4 that for $r_0 > \lambda$ and $\theta > 50^\circ$

$$\frac{E'(\theta)-E_1(\theta)}{E_1(\pi/2)} \simeq \frac{\mathfrak{z}}{\eta}\frac{K(r_0)}{\sin\theta}\left[e^{-j\beta r_0}J_0(\beta r_0 \sin\theta) - \sec\theta + J_s(\sin\theta, \beta r_0) + jJ_c(\sin\theta, \beta r_0)\right] \tag{11}$$

The functions $J_c(\mu, x)$ and $J_s(\mu, x)$ have been defined by Schwarz* as

$$\left.\begin{aligned} J_c(\mu, x) &= \int_0^x J_0(\mu u)\cos u \, du \\ J_s(\mu, x) &= \int_0^x J_0(\mu u)\sin u \, du \end{aligned}\right\} \tag{12}$$

Schwarz's paper includes tables for values of x up to 5. These were extended to $x = 2\pi$ by numerical integration.

If the factor $K(r_0)$ were omitted, the solution would depend upon the assumption that the total surface current over perfectly-conducting ground is constant in magnitude and retarded in phase according to the velocity of light at all distances greater than r. In other words, $K(r_0)$ provides a correction for the contribution of the induction field to the total surface current. It is given by

$$K(r_0) = \frac{I_1(r_0)e^{j\beta r_0}}{\underset{r\to\infty}{Lt}\left\{I_1(r)e^{j\beta r}\right\}} \tag{13}$$

Its value for an aerial carrying a sinusoidal current distribution is given in Appendix 7.4.

In spite of the factor cosec θ on the right-hand side of (11), the expression tends to zero with θ, being proportional to it for small values.

Equation 11 has been used to compute vertical radiation patterns in a case that had already been investigated experimentally by means of small-scale models[4, 5]: a 0·55 λ aerial with a perfectly-conducting earth system one wavelength in radius. In the experiments, imperfectly conducting ground was simulated by solutions of salt in water having relative permittivities of 80-j180 and 80-j36, corresponding respectively to highly-conducting ground (10^{-2}mhos/m) and poorly-conducting ground (2×10^{-3} mhos/m) at 1 Mc/s. Experimental and theoretical results are compared in Figs. 14 and 15.

In each figure the six curves (a)-(f) were obtained as follows:

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*The reference to Schwarz's paper[14] was provided by Dr. G.G. MacFarlane.

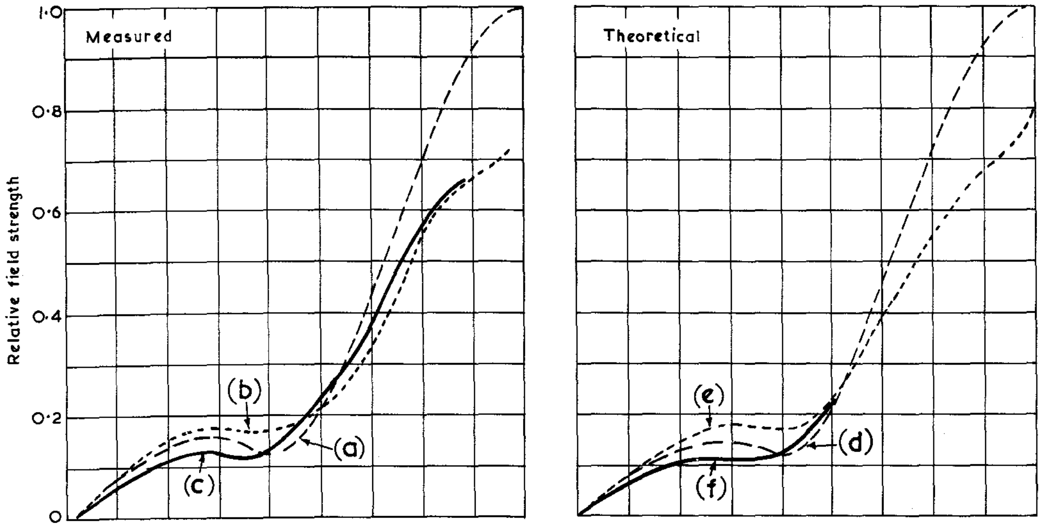

Fig. 14 - The effect of a perfectly-conducting earth system one wavelength in radius on the vertical radiation pattern of a 0·55 λ vertical aerial. Complex relative permittivity 80-j180.

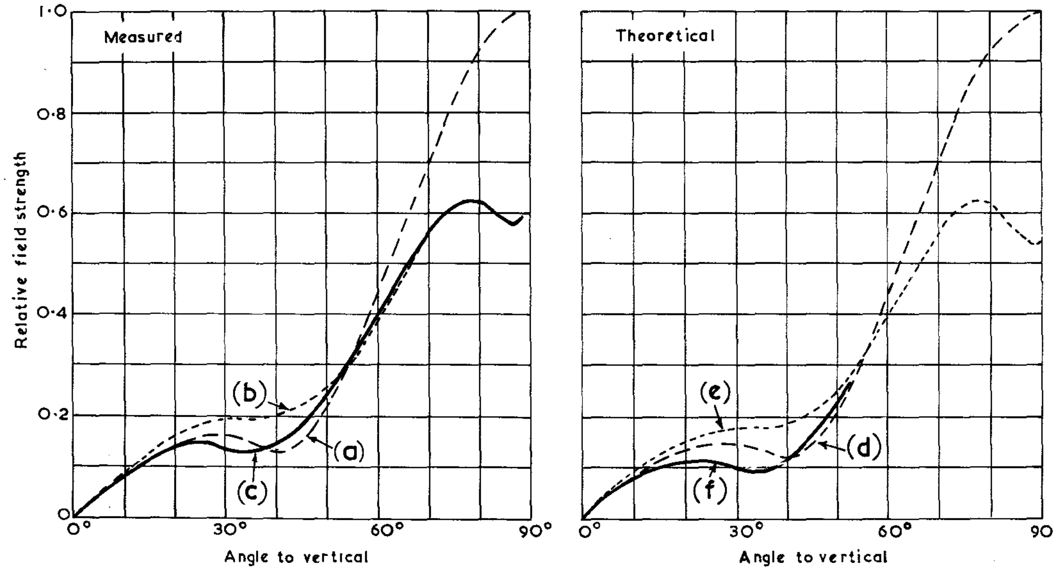

Fig. 15 - The effect of a perfectly-conducting earth system one wavelength in radius on the vertical radiation pattern of a 0·55 λ vertical aerial. Complex relative permittivity 80-j36.

— — — — — Perfectly-conducting ground.

- - - - - - - - - - - Imperfectly-conducting ground.

————— Imperfectly-conducting ground with earth system.

a. Measured, perfectly-conducting ground.

From measurements on a symmetrical dipole having a physical length of 0·55 λ and a characteristic impedance of 245 ohms for each half (E.036[8], Fig. 6).

b. Measured, imperfectly-conducting ground.

From measurements on a model aerial over a salt-water surface at a distance of 9·4 wavelengths. The finite distance was not expected to affect the radiation pattern at angles of less than 70° to the vertical. Field strength scaled for best fit with curve (e). (E.041[4] Fig. 10, and E.043[5] Fig. 3.)

c. Measured, imperfectly-conducting ground with earth system.

Measured in the same way as (b). The earth-system was a sheet-metal disc, level with the liquid surface. The vertical scale factor was chosen arbitrarily. (E.041[4] Fig. 15, and E.043[5] Fig. 6.)

d. Theoretical, perfectly-conducting ground.

Böhm's feed current taken into account and a velocity factor of 0·92 assumed. (E.036, Fig. 6.)

e. Theoretical, imperfectly-conducting ground.

Same current distribution assumed as for (d). Finite distance taken into account (affecting angles of more than 70° to the vertical only). Field strength scaled so that the same current over perfectly-conducting ground would give unit field in horizontal directions. (E.041[4] Fig. 10, and E.043[5] Fig. 3.)

f. Theoretical, imperfectly-conducting ground with earth system.

From Equation 11, which gave the departure from curve (d). Infinite distance assumed. Field strength scaled so that the same current over perfectly-conducting ground would give unit field in horizontal directions.

As pointed out previously, the vertical scale for curve (c) was chosen arbitrarily. If it is reduced slightly the agreement between the theoretical and experimental results is found to be quite good.

5. CONCLUSIONS.

In practice the importance of the earth system at a medium-wave transmitting station resides solely in its effect on the efficiency of the radiating system. Earth systems extensive enough to effect a useful improvement in the vertical radiation pattern would not be practicable in the United Kingdom.

Efficiency is difficult to measure accurately, and direct experimental evidence in support of the theoretical results is therefore lacking at present. Nevertheless, the experiments at Barnstaple have confirmed the general order of magnitude of the predicted increase in efficiency resulting from an extension of the earth system. Moreover an extension of the theory to the vertical radiation pattern has given results which, though not themselves of great practical importance, have enabled the method to be checked experimentally: the agreement is considered to be satisfactory.

The principal results are contained in Figs. 11 and 12, which show the effect of the radius of the earth system on the efficiency. Although only three heights of aerial are considered, it should be possible to decide upon the earth system required in most cases.

It is, of course, always beneficial to erect a transmitting aerial on a site of high conductivity, say 10^{-2} mhos/m. In this case, at low-power stations where the effective height of the aerial is between 0·1 and 0·25 wavelengths, there is little to be gained by installing an earth system more than 0·2 wavelengths in radius. Should it be possible to reduce the area of the site required, by curtailing the length of some of the earth wires to (say) 0·1 wavelengths, the saving of land would justify the small loss of efficiency. The best compromise between cost and efficiency is obtained with a system of about 72 wires; this conclusion is based on the results of an earlier investigation into the effect of earth systems on the impedance of short aerials (E.042/2, Fig. 22).

For sites of low conductivity (10^{-3} mhos/m), an earth system 0·3 wavelengths in radius is sufficient.

At high-power stations with antifading mast radiators it has been usual in the B.B.C. to instal an earth system about one half-wavelength in radius. Fig. 2 shows that this radius is unnecessarily great on a site of high conductivity; there is little to be gained by a radius greater than 0·2 wavelengths. A radius of 0·4 wavelengths is desirable for a site of low conductivity, particularly if loop feeding or double feeding is adopted. Nevertheless, if the site is larger than sufficient for an earth system 0·2 wavelengths in radius (this might be required so as to include stay anchorages), it would be desirable to extend the earth wires to the boundary. This would be advantageous if the aerial were used for long-wave broadcasting at some later date.

This report concludes a general theoretical investigation into medium-wave earth systems. In most cases it will be sufficient to use the results given here as a basis for a decision, even though they may not apply exactly to the problem under consideration. Should it nevertheless become essential to carry out further complete calculations, the labour would be reduced by using the absorbed-energy method for computing relative efficiencies.

6. REFERENCES

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7. APPENDICES.

7.1. Derivation of Equation 3.

This derivation avoids certain theoretical objections to which the derivation in the text may be open. It is also capable of extension to systems lacking circular symmetry (e.g. semicircular earth systems——see Appendix 7.3).

Let the aerial under consideration be denoted by A, and let its effective height (i.e. the line integral of current along its length divided by the input current) be h. It will be supposed that a second vertical aerial B, which may be regarded as a device for measuring the field of aerial A, is erected at some distant point (Fig. 16).

In the absence of an extensive earth system, let a current I_0 impressed between the terminals of the aerial A result in a magnetic intensity whose component tangential to the surface of the ground is $\underline{H}_{At}$. Let the tangential magnetic intensity resulting from a current I_0 impressed between the terminals of the aerial B be $\underline{H}_{Bt}$. Let the mutual impedance between the aerials be Z_{AB}.

Now suppose that, within some limited area S surrounding A, the surface impedance of the ground is changed from $\mathfrak{z}$ to $\mathfrak{z}'$, and that in consequence $\underline{H}_{At}$ and Z_{AB} change to $\underline{H}'_{At}$ and Z'_{AB}. It has been shown[3] that

$$Z'_{AB}-Z_{AB} \quad = \quad (1/I_0^2)\iint_S (\mathfrak{z}'-\mathfrak{z})\underline{H}'_{At}\circ\underline{H}_{Bt}\ dS \qquad (14)$$

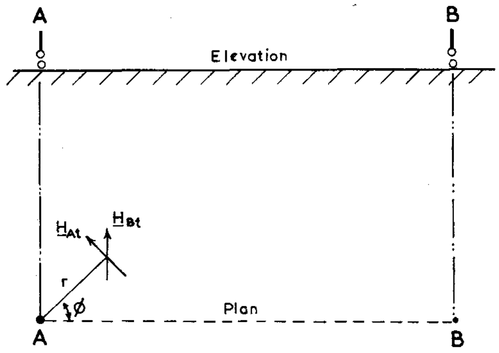

Fig. 16

If B is sufficiently distant from A, then, at all points within S, $\underline{H}_{Bt}$ will be perpendicular to AB and constant in magnitude; its phase will vary from point to point in a manner corresponding to propagation with the velocity of light. Taking polar coordinates (r,ϕ) as shown in Fig. 16, we may write

$$\underline{H}'_{At}\circ\underline{H}_{Bt} \quad = \quad H'_{At}H_{Bt0}e^{j\beta r\cos\phi}\cos\phi \qquad (15)$$

where H_{Bt0} is the value (complex) of H_{Bt} at the point A. Now the component of electric intensity associated with H_{Bt0} will at A be equal to ηH_{Bt0} (directed vertically upwards)*. Hence

$$I_0Z_{AB} \quad = \quad -h\eta H_{Bt0} \qquad (16)$$

Substituting in (15),

$$\underline{H}'_{At}\circ\underline{H}_{Bt} \quad = \quad -\frac{I_0Z_{AB}}{h\eta}H'_{At}e^{j\beta r\cos\phi}\cos\phi \qquad (17)$$

Writing $I'(r)/2\pi r$ for H'_{At}, substituting in (14), and expressing dS in polar coordinates, this equation becomes

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*This is not exact because of wave tilt, but the error is negligible.

$$\frac{Z'_{AB}-Z_{AB}}{Z_{AB}} = -\frac{1}{2\pi I_0 h\,\eta}\iint\limits_{S}(\mathfrak{z}'-\mathfrak{z})I'(r)e^{j\beta r\cos\phi}\cos\phi\;dr\;d\phi \tag{18}$$

For a circularly-symmetrical earth system of radius r_0 (18) becomes

$$\frac{Z'_{AB}-Z_{AB}}{Z_{AB}} = -\frac{1}{2\pi I_0 h\,\eta}\int_0^{r_0}(\mathfrak{z}'-\mathfrak{z})I'(r)\left[\int_0^{2\pi} e^{j\beta r\cos\phi}\cos\phi\;d\phi\right]dr \tag{19}$$

The integral within the square brackets is equal to $2\pi\,jJ_1(\beta r)$. Hence

$$\frac{Z'_{AB}-Z_{AB}}{Z_{AB}} = \frac{-j}{I_0 h\,\eta}\int_0^{r_0}(\mathfrak{z}'-\mathfrak{z})I'(r)J_1(\beta r)\;dr \tag{20}$$

Since the mutual impedance between the aerials is a measure of the groundwave field of Aerial A at B, Z_{AB} and Z'_{AB} can be replaced by E and E', giving Equation 3.

7.2. Graphical Computation Procedure.

Most of the labour is involved in the computation of the integral in Equation 4. The first step is to estimate the real and imaginary parts of $(\mathfrak{z}-\mathfrak{z}')I_1(r)$ for each combination of aerial and earth system to be considered. The problem is therefore to compute a number of integrals of the form

$$\int_0^{r_0} f(r)J_1(\beta r)dr$$

Since $f(r)$ and $J_1(\beta r)$ are oscillatory functions their product varies in rather a complicated manner, and a large number of ordinates would be required for direct integration.

The difficulty is overcome by taking out the factor $J_1(\beta r)$, which is common to all cases, by changing the variable

$$\int_0^{r_0} f(r)J_1(\beta r)\;dr = -\frac{1}{\beta}\int_{r=0}^{r_0} f(r)d\{J_0(\beta r)\} \tag{21}$$

The integration is then performed graphically, using graph paper marked with a horizontal scale corresponding to $J_0(\beta r)$. The fact that this scale is "folded" causes no particular difficulty. For obtaining the results shown in this report, an

"Albrit Integrator" was used.

It may be remarked that the use of non-uniform scales to facilitate graphical integration has a wide field of application. When computing changes in efficiency by the absorbed-energy method it is convenient to use a logarithmic radius scale, since with a uniform scale, the factor $1/r$ in the integrand of (11) causes difficulty at small radii. Graph paper with a $(1-\cos\theta)$ horizontal scale and a square-law vertical scale is used in this Department for calculating the power gain of omnidirectional aerials from their vertical radiation patterns.

7.3. Derivation of Equation 5.

Consider firstly a semicircular earth system oriented so that the bisecting radius lies along the path of propagation AB (Fig. 8a). The only amendment to Appendix 7.2 required is to restrict the integration with respect to ϕ to the range $-\pi/2$ to $+\pi/2$. Thus for

$$\int_0^{2\pi} e^{j\beta r\cos\phi}\cos\phi\, d\phi = 2\pi\, jJ_1(\beta r)$$

we must substitute

$$\int_{-\pi/2}^{\pi/2} e^{j\beta r\cos\phi}\cos\phi\, d\phi = 2\pi\, jF(\beta r)$$

Expanding the exponential, as in frequency modulation theory,

$$F(\beta r) = \frac{1}{2\pi j}\int_{-\pi/2}^{\pi/2}\left\{J_0(\beta r) + 2\sum_1^{\infty} j^n J_n(\beta r)\cos n\phi\right\}\cos\phi\, d\phi$$

$$= \frac{1}{2}J_1(\beta r) - \frac{j}{\pi}\left\{J_0(\beta r) - 2\sum_1^{\infty}\frac{J_{2m}(\beta r)}{4m^2-1}\right\}$$

which is Equation 5.

7.4 Derivation of Equation 11.

In Fig. 17 A represents the aerial under consideration; this will be supposed to have an effective height h. The reference aerial B, which may be regarded as a dipole, is now elevated to an angle θ to the vertical. In the first place it will be supposed that the ground is everywhere perfectly conducting. Let the components of magnetic intensity tangential to the surface of the ground, resulting from a current I_0 impressed between the terminals of aerials A, B, be $\underline{H}_{At}$, $\underline{H}_{Bt}$, respectively. Let the mutual impedance between the aerials be Z_{AB}*.

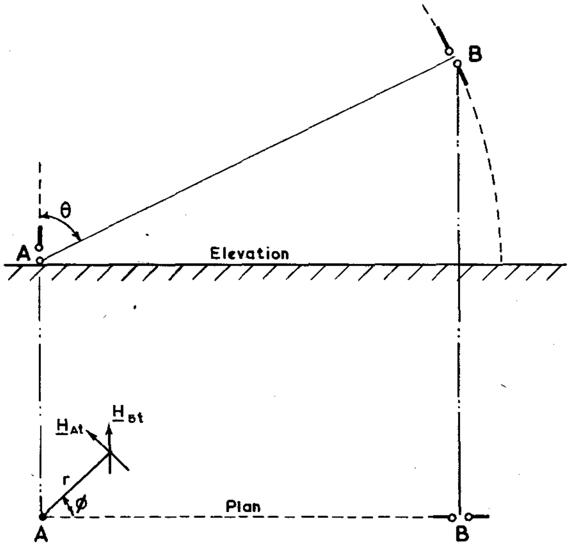

Fig. 17

Now suppose that within an area S the ground conductivity is changed from zero to $\mathfrak{z}$, and that in consequence $\underline{H}_{At}$, $\underline{H}_{Bt}$, Z_{AB} change to $\underline{H}'_{At}$, $\underline{H}'_{Bt}$, Z'_{AB}. Then the generalised form of the compensation theorem gives

$$Z'_{AB}-Z_{AB} = (1/I_0^2\iint_S \mathfrak{z}\,\underline{H}'_{At}\circ\underline{H}_{Bt}\,dS \qquad (22)$$

This corresponds to (14) of Appendix 7.1.

Assuming that the distance AB is large compared with the wavelength, and with the distance between A and any part of S contributing appreciably to the integral, the magnitude of H_{Bt} may be regarded as constant. The angle between $\underline{H}'_{At}$ and $\underline{H}_{Bt}$ will be equal to ϕ. The phase of H_{Bt} will vary from point to point of S in conformity with a horizontal phase velocity c cosec θ, where c is the velocity of light. It follows that

$$\underline{H}'_{At}\circ\underline{H}_{Bt} = H'_{At}\,H_{Bt0}\cos\phi\,e^{j\beta r\sin\theta\cos\phi} \qquad (23)$$

where H_{Bt0} is the complex value of H_{Bt} at the point A ($r = 0$).

H_{Bt0} is independent of θ. It is convenient to express it in terms of $Z_{AB}(\pi/2)$, the mutual impedance, over perfectly-conducting ground, when $\theta = \pi/2$. Consider, therefore, the condition of perfectly-conducting ground with θ equal to $\pi/2$. The electric intensity associated with H_{Bt0} at A will be equal to ηH_{Bt0}, and will be directed vertically upwards. Hence

$$Z_{AB}(\pi/2) = -\eta h H_{Bt0}/I_0 \qquad (24)$$

Using (23) and (24), and writing $I'(r)/2\pi r$ for H'_{At}, (22) becomes

$$Z'_{AB}(\theta) - Z_{AB}(\theta) = -\frac{Z_{AB}(\pi/2)}{2\pi I_0\eta h}\iint_S \mathfrak{z}\,I'(r)e^{j\beta r\sin\theta\cos\phi}\cos\phi\,(1/r)\,dS \qquad (25)$$

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*In order to conform with the convention adopted elsewhere in this report, field components and mutual impedances corresponding to perfectly-conducting ground should be distinguished by the subscript 1. For convenience this has been dropped in this Appendix.

Taking S to be the area outside a perfectly-conducting circular earth system, radius r_0, (25) becomes

$$\frac{Z'_{AB}(\theta)-Z_{AB}(\theta)}{Z_{AB}(\pi/2)} = -\frac{\mathfrak{z}}{2\pi I_0 \eta h}\int_{r_0}^{\infty} I'(r)\left[\int_0^{2\pi} e^{j\beta r \sin\theta\cos\phi}\cos\phi \, d\phi\right] dr$$

$$= -\frac{j\mathfrak{z}}{I_0 \eta h}\int_{r_0}^{\infty} I'(r) J_1(\beta r \sin\theta)\, dr \qquad (26)$$

At this stage an approximation must be made, since $I'(r)$, the total surface current when the ground outside the earth system is imperfectly-conducting, is not known exactly. Now it can be shown that the relation

$$\int_a^{\infty} F(x) J_1(\mu x) e^{-jx} dx \simeq F(a)\int_a^{\infty} J_1(\mu x) e^{-jx} dx \qquad (27)$$

is approximately true for $0 < \mu < 1$ and $a > 2\pi$, provided that $F(x)$ varies sufficiently slowly with x, so that it is approximately the same for values of x differing by $1/(1-\mu)$. Provided that $\beta r_0 > 2\pi$, and that θ is not too close to $\pi/2$*, (27) enables $I'(r)$ to be approximated by

$$I(r_0) e^{j\beta(r_0 - r)}$$

A study of the effects of groundwave attenuation and the induction field on $I'(r)$ indicates that the result will be sufficiently accurate for values of θ less than 50°. Equation 26 becomes

$$\frac{Z'_{AB}(\theta)-Z_{AB}(\theta)}{Z_{AB}(\pi/2)} \simeq -\frac{j\mathfrak{z} I(r_0)}{I_0 \eta h} e^{j\beta r_0}\int_{r_0}^{\infty} e^{-j\beta r} J_1(\beta r \sin\theta) dr \qquad (28)$$

Integrating by parts, the integral can be expressed in terms of the functions defined in Equation 12. The result is

$$\frac{Z'_{AB}(\theta)-Z_{AB}(\theta)}{Z_{AB}(\pi/2)} \simeq -\frac{j\mathfrak{z} I(r_0) e^{j\beta r_0}}{\eta I_0 \beta h \sin\theta}\Big[e^{-j\beta r_0} J_0(\beta r_0 \sin\theta) - \sec\theta + J_s(\sin\theta, \beta r_0) + jJ_c(\sin\theta, \beta r_0)\Big] \qquad (29)$$

---

*Hence the method cannot be used to find the groundwave attenuation factor.

If the induction field could be neglected, $I(r_0)$ would be equal to

$$j\beta h I_0 e^{-j\beta r_0}.$$

Writing

$$I(r_0) \quad = \quad j\beta h I_0 e^{-j\beta r_0} K(r_0), \tag{30}$$

where K is a correcting factor, (29) becomes

$$\frac{Z'_{AB}(\theta) - Z_{AB}(\theta)}{Z_{AB}(\pi/2)} \simeq \frac{\mathfrak{z} K(r_0)}{\eta \sin\theta} \left[e^{-j\beta r_0} J_0(\beta r_0 \sin\theta) - \sec\theta \right.$$

$$\left. + J_s(\sin\theta, \beta r_0) + jJ_c(\sin\theta, \beta r_0) \right] \tag{31}$$

Since the mutual impedance between the aerials is a measure of the field of aerial A at B, Z_{AB} and Z'_{AB} can be replaced by E and E', giving Equation 11.

In the case of an idealised aerial with a sinusoidal current distribution, for which $I(r)$ is given in Equation 4 of E.042, $K(r)$ is given by

$$K(r) \quad = \quad \frac{1}{1-\cos\beta h} \left[e^{-j\beta\{\sqrt{r^2+H^2}-r\}} - \cos\beta h \right] \tag{32}$$

where H is the physical height of the aerial.